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ANALYSIS OF EFFECTS OF SPANWISE VARIATIONS OF GUST VELOCITY ON A VANE-CONTROLLED GUST-ALLEVIATION SYSTEM

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16. Abstract

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SUMMARY

An analysis has been made of the effects of spanwise variations of gust velocity in isotropic turbulence on a gust-alleviation system which employs an angle-of-attack vane mounted ahead of the wing to sense the vertical gust velocity. The wing flaps were moved in response to the vane deflection by a linear second-order servosystem to produce a lift opposite to that produced by the gust. Since the gust velocity actually varies across the wing span and the vane only senses the gust velocity at a single point, the flaps cannot be expected to eliminate the effects of the gusts completely by being deflected in response to a signal from the vane. The effectiveness of the alleviation system is examined by considering the statistical properties of the residual lift on the alleviated wing. Consideration of spanwise variation of gusts has indicated design parameters (gain and natural frequency of the flap servosystem and vane location) that give substantial reduction of the lift due to gusts.

INTRODUCTION

To insure passenger comfort and reduce structural loads when an airplane flies through rough air, it is desirable to minimize resulting accelerations. In 1951, a rather thorough theoretical analysis of some methods for increasing the smoothness of flight through rough air was made. (See ref. 1.) As a result of this analysis, a method of gust alleviation by direct lift control evolved and was later flight tested. (See refs. 2 and 3.) Although the gust alleviation was substantial in the flight tests, the system did not work as well as had been anticipated.

With the increasing emphasis in space projects, this gust alleviation project was discontinued. Also, other reasons why there was less interest in gust alleviation at this time are (1) jet transports tended to fly at high altitudes which reduced the frequency of gust encounters, (2) higher wing loading was used, and (3) improved weather forecasting and the ability to locate storm centers permitted aircraft to avoid turbulent air.

Interest in gust-load alleviation has recently begun to increase because of possible applications to V/STOL aircraft. This in turn has called attention to the flight tests mentioned previously.

There are several possible reasons why less gust-load alleviation was obtained in the flight tests (refs. 2 and 3) than had been predicted in reference 1. Three of these are (1) the effect of spanwise gust variation, (2) inadequate or nonlinear servo response, and (3) nonlinear flap effectiveness. In the present study the reduction in the wing lift due to turbulence is examined with the spanwise variation of the gusts taken into account. Some results are also included of the desired characteristics of the flap servomechanism.

SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

b	wing span
c	wing chord
$\bar{c}$	wing mean aerodynamic chord
C	nondimensional gust frequency, $\frac{\omega_g L}{U}$
C_L	lift coefficient
c_l	local lift coefficient
C_{L_α}	lift-curve slope, $\frac{\partial C_L}{\partial \alpha}$
c_{l_α}	local lift-curve slope, $\frac{\partial c_l}{\partial \alpha}$
$C_{L_{\delta_f}}$	change in wing lift coefficient with flap deflection, $\frac{\partial C_L}{\partial \delta_f}$
e	lift on the alleviated wing due to vertical gusts, $L_w - L_f$
g	nondimensional correlation function for vertical gust
$h(x)$	impulse-response function of the flap system

K	gain constant, $\frac{C_{L_{\delta_f}}}{C_{L_\alpha}} K_1$
K_1	gain constant in flap control system
k	nondimensional natural frequency of flap servo, $\frac{\omega_n L}{U}$
L	scale of turbulence
l	vane location forward of aerodynamic center of the wing
l^*	nondimensional vane location, $\frac{l}{b}$
L_f	lift on wing due to flap deflection
L_w	lift on wing due directly to vertical gusts
q	dynamic pressure
R	percentage reduction in the power spectrum of the basic wing when the alleviation system is used, $\frac{\phi_1 - \tilde{\phi}_e}{\phi_1} \times 100$
r	percentage reduction in mean-square lift on basic wing when the alleviation system is used, $\frac{\psi_1(0) - \tilde{\psi}_e(0)}{\psi_1(0)} \times 100$
S	wing surface area
t	time
U	constant forward velocity of the wing
w	vertical gust velocity
X,Y	reference axes
x,y	coordinates
y^*	nondimensional spanwise coordinate, $\frac{y}{b/2}$

α	angle of attack on the wing
α_g	angle of attack due to vertical gust velocity
α_v	angle of attack of vane due to vertical gust
γ	lift weighting function
$\delta(x)$	Dirac delta function
δ_f	flap deflection
ζ	damping ratio in flap control system
λ	dummy variable of integration
ξ	nondimensional distance along flight direction, $\frac{U\tau}{L}$
σ	distance between two points in space
τ	time increment for wing traveling at constant velocity U to travel from x_1 to x_2
ϕ_e	power spectrum of $e(x)$
$\tilde{\phi}_e$	nondimensional power spectrum of $e(x)$
ϕ_1	nondimensional power spectrum of L_w
ψ_e	correlation function of $e(x)$
$\tilde{\psi}_e$	nondimensional correlation function of $e(x)$
ψ_w	correlation function of vertical gust
ψ_1	nondimensional correlation function of L_w
Ω_n	spatial natural frequency of flap servo in radians per unit of distance

ω_g gust frequency

ω_n natural frequency of flap servo

All primed values denote nondimensionalization by the scale of turbulence L. For example, $b' = \frac{b}{L}$. The symbol Δ preceding a quantity means an incremental value. For example, $\Delta x = x_2 - x_1$.

STATEMENT OF PROBLEM

The problem being examined is illustrated in figure 1. The airplane is assumed to be flying in a straight line with wing level through gusty air which varies not only longitudinally but also across the wing span of the airplane. Based upon the gust sensed at a point by a small vane, the flaps are deflected either up or down as required to alleviate the effect of the gust. In some cases, the flaps were assumed to deflect in phase with the vane, and in others, the flaps were assumed to be operated by a second-order servomechanism. Note that the gust velocity hitting the vane is not necessarily equal to the gust velocities at various points along the span. For this reason, the flap system cannot be expected to eliminate the gust load totally if it responds to the vane deflection. A measure of the residual lift on the wing can, however, be estimated by using the statistical properties of gusts to calculate the statistical properties of the residual lift.

A method for computing the statistical properties of lift on an airplane wing due to random atmospheric turbulence is given in reference 4. The crux of the approach is to consider the turbulence isotropic so that the statistical characteristics of the turbulence are independent of direction. In this manner, the statistical properties of the vertical

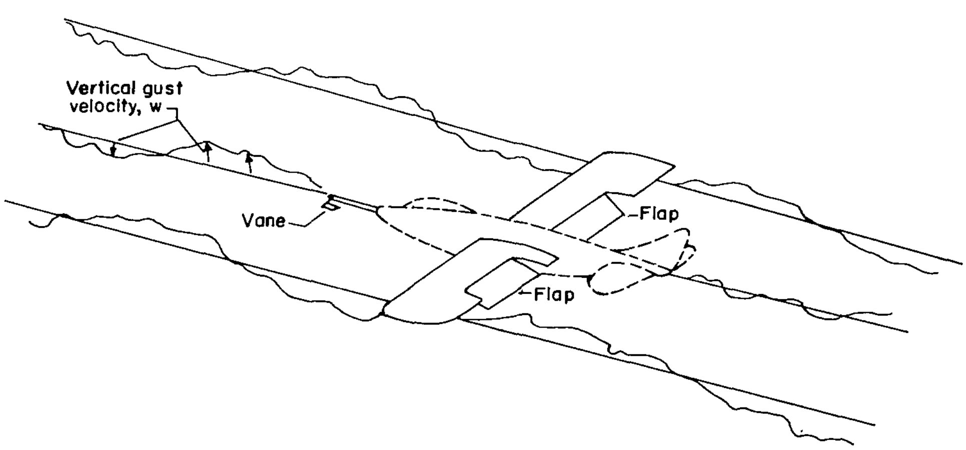


Figure 1.- Airplane flying through isotropic turbulence.

gust velocities along the wing span can be related to those of the vertical gust velocities along the flight path, and consequently, an analytical solution to the problem can be formulated.

The approach of reference 4 is used in the present study to examine the statistical properties of lift on a wing due to vertical gusts when the wing is equipped with an alleviation system.

ANALYSIS

The coordinate system used in this study is shown in figure 2. A rigid wing is moving along the X-direction with constant forward velocity U. A vane is mounted a distance l ahead of the aerodynamic center to sense the gust.

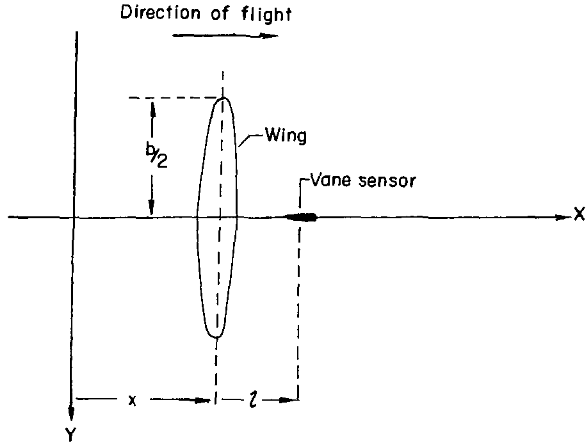


Figure 2.- Coordinate system used in this study.

Lift on the Wing-Flap System

A theorem in linearized aerodynamics states that the lift in steady or indicial motion of a wing having arbitrary twist and camber is equal to the integral over the planform of the product of the local angle of attack and the loading per unit angle of attack at the corresponding point of a flat-plate wing of identical planform in reverse flow (ref. 5, pp. 325-326). The wing lift, therefore, can be written as

$$L_W(x) = \frac{b}{2}\int_{-1}^{1} qc\left(c_l\right)_{\text{unit }\alpha}\ \alpha(x,y^*)dy^* \tag{1}$$

The local lift coefficient per unit angle of attack of the wing is equal to the local section lift-curve slope; therefore, equation (1) can be written as

$$L_w(x) = q \frac{b}{2} \int_{-1}^{1} cc_{l_\alpha} \alpha(x,y^*)dy^* \tag{2}$$

Multiplying and dividing through by the product of wing lift-curve slope and wing area results in

$$L_w(x) = \frac{qSC_{L_\alpha}}{2} \int_{-1}^{1} \frac{cc_{l_\alpha}}{\bar{c}C_{L_\alpha}} \alpha(x,y^*)dy^* \tag{3}$$

Since the local section lift-curve slope and the total wing lift-curve slope are based on the same angle of attack, equation (3) can be written as

$$L_w(x) = \frac{qSC_{L_\alpha}}{2} \int_{-1}^{1} \frac{cc_l}{\bar{c}C_L} \alpha(x,y^*)dy^* \tag{4}$$

or

$$L_w(x) = \frac{qSC_{L_\alpha}}{2} \int_{-1}^{1} \gamma(y^*)\alpha(x,y^*)dy^* \tag{5}$$

The angle of attack at location (x,y) caused by a vertical gust is given approximately by

$$\alpha_g(x,y) = \frac{w(x,y)}{U} \tag{6}$$

The wing lift caused by a vertical gust which varies across the span is therefore given by

$$L_w(x) = \frac{qSC_{L_\alpha}}{U} \frac{1}{2} \int_{-1}^{1} \gamma(y^*)w(x,y^*)dy^* \tag{7}$$

If the wing is provided with a flap, the increment in lift caused by flap deflection can be expressed by

$$L_f(x) = qSC_{L_{\delta_f}} \delta_f(x) \tag{8}$$

In this analysis it is assumed that the flap deflection $\delta_f(x)$ is the response of a linear system, with angle of attack of the sensing vane $\alpha(x + l, 0)$ as the input variable; therefore, the flap deflection is given by the familiar Duhamel convolution integral as

$$\delta_f(x) = K_1 \int_0^{\infty} h(\lambda)\alpha_v(x + l - \lambda, 0)d\lambda \tag{9}$$

where $h(\lambda)$ is the flap response due to a unit impulsive vane angle of attack, and $h(\lambda) = 0$ for $\lambda < 0$. In the section "Application of Equations," expressions will be introduced for

$h(\lambda)$ which correspond to the following cases: (1) the flap moving in phase with the vane deflection and (2) the flap deflection related to the vane deflection through a linear system of second order.

By assuming no lag in the response of the vane to a gust, the angle of attack of the vane can be expressed as

$$\alpha_v(x+l,0) = \frac{w(x+l,0)}{U} \tag{10}$$

Substituting equations (9) and (10) into equation (8) results in

$$L_f(x) = K_1 \frac{qSC_{L_{\delta_f}}}{U} \int_0^\infty h(\lambda)w(x+l-\lambda,0)d\lambda \tag{11}$$

In the gust-alleviation concept of this paper, the flap is used to reduce the lift caused by the gust. The lift on the alleviated wing due to vertical gusts therefore is given by

$$e(x) = L_w(x) - L_f(x)$$

or, from equations (7) and (11),

$$e(x) = \frac{qSC_{L_\alpha}}{U}\left[\frac{1}{2}\int_{-1}^{1} \gamma(y^*)w(x,y^*)dy^* - K\int_0^\infty h(\lambda)w(x+l-\lambda,0)d\lambda\right] \tag{12}$$

Statistical Properties of Lift on a Wing Due to Vertical Gusts

The statistical properties of interest are the correlation function and power spectrum of the lift on the wing-flap system. These functions depend somewhat on the assumed model of atmospheric turbulence. In this study, it is assumed that the turbulence is stationary, homogeneous, and isotropic. Under these assumptions the correlation function between vertical gust velocities at two points depends only on the distance between the points, and not on their location in space. The correlation function, therefore, can be expressed by

$$\begin{aligned}\psi_w(x_2-x_1,y_2-y_1) &= \lim_{X\to\infty} \frac{1}{2X}\int_{-X}^{X} w(x_1,y_1)w(x_2,y_2)dx_1 \\ &= \overline{w^2}g\left[\sqrt{(\Delta x)^2+(\Delta y)^2}\right] \\ &= \overline{w^2}g(\sigma) \end{aligned} \tag{13}$$

where $\overline{w^2}$ is the mean-square vertical gust velocity, and $g(\sigma)$ is a nondimensional correlation function for vertical gusts which is a scalar function of the distance between the two points (ref. 6).

Correlation function of lift.- The correlation function for the resultant lift on the wing at points x_1 and x_2 along the direction of flight is defined by

$$\psi_e(x_2-x_1) = \lim_{X\to\infty} \frac{1}{2X}\int_{-X}^{X} e(x_1)e(x_2)dx_1 \tag{14}$$

where $x_2 = x_1 + \Delta x$. Substitution of equation (12) into equation (14) and then making use of the relation shown in equation (13) results in

$$\begin{aligned}\psi_e(x_2-x_1) = \left(\frac{qSC_{L_\alpha}}{U}\right)^2 \overline{w^2}\Bigg\{\frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} \gamma(y_1^*)\gamma(y_2^*)g\left[\sqrt{(\Delta x)^2 + \left(\frac{b}{2}\right)^2(\Delta y^*)^2}\right]dy_1^*\, dy_2^* \\ - \frac{K}{2}\int_{-1}^{1}\int_{0}^{\infty} \gamma(y_1^*)h(\lambda_2)g\left[\sqrt{(\Delta x + l - \lambda_2)^2 + \left(\frac{b}{2}\right)^2(y_1^*)^2}\right]d\lambda_2\, dy_1^* \\ - \frac{K}{2}\int_{-1}^{1}\int_{0}^{\infty} \gamma(y_2^*)h(\lambda_1)g\left[\sqrt{(\Delta x - l + \lambda_1)^2 + \left(\frac{b}{2}\right)^2(y_2^*)^2}\right]d\lambda_1\, dy_2^* \\ + K^2\int_{0}^{\infty}\int_{0}^{\infty} h(\lambda_1)h(\lambda_2)g\left(\left|\Delta x - \Delta\lambda\right|\right)d\lambda_1\, d\lambda_2\Bigg\}\end{aligned} \tag{15}$$

It is assumed that correlations in space and time are equivalent so that $x_2 - x_1$ can be replaced by $U\tau$ in equation (15). This assumption is a result of Taylor's hypothesis. (See ref. 6.) For later convenience this substitution is made and equation (15) is written in the following form:

$$\tilde{\psi}_e(U\tau) = \psi_1(U\tau) - K\psi_2(U\tau) - K\psi_3(U\tau) + K^2\psi_4(U\tau) \tag{16}$$

where

$$\tilde{\psi}_e(U\tau) = \frac{\psi_e(U\tau)}{\left(\frac{C_{L_\alpha}qS}{U}\right)^2\overline{w^2}} \tag{17}$$

$$\psi_1(U\tau) = \frac{1}{4}\int_{-1}^{1}\int_{-1}^{1} \gamma(y_1{}^*)\gamma(y_2{}^*)g\left[\sqrt{(U\tau)^2 + \left(\frac{b}{2}\right)^2(\Delta y^*)^2}\right]dy_1{}^*\,dy_2{}^* \tag{18}$$

$$\psi_2(U\tau) = \frac{1}{2}\int_{-1}^{1}\int_{0}^{\infty} \gamma(y_1{}^*)h(\lambda_2)g\left[\sqrt{(U\tau + l - \lambda_2)^2 + \left(\frac{b}{2}\right)^2(y_1{}^*)^2}\right]d\lambda_2\,dy_1{}^* \tag{19}$$

$$\psi_3(U\tau) = \frac{1}{2}\int_{-1}^{1}\int_{0}^{\infty} \gamma(y_2{}^*)h(\lambda_1)g\left[\sqrt{(U\tau - l + \lambda_1)^2 + \left(\frac{b}{2}\right)^2(y_2{}^*)^2}\right]d\lambda_1\,dy_2{}^* \tag{20}$$

$$\psi_4(U\tau) = \int_{0}^{\infty}\int_{0}^{\infty} h(\lambda_1)h(\lambda_2)g\left(\left|U\tau - \Delta\lambda\right|\right)d\lambda_1\,d\lambda_2 \tag{21}$$

Mean-square value of lift.- The mean-square value of lift is obtained by averaging the square of the lift $e(x)$ throughout the region of turbulence, which is precisely the value obtained by evaluating equation (14) with $x_1 = x_2$, that is, $\psi_e(0)$. The nondimensional mean-square value of $e(x)$, therefore, can be written as (from eq. (16))

$$\tilde{\psi}_e(0) = \psi_1(0) - K\psi_2(0) - K\psi_3(0) + K^2\psi_4(0) \tag{22}$$

From the definitions of ψ_2 and ψ_3 given by equations (19) and (20), it follows that when Δx (or $U\tau$) is equal to zero,

$$\psi_2(0) = \psi_3(0)$$

and equation (22) reduces to

$$\tilde{\psi}_e(0) = \psi_1(0) - 2K\psi_2(0) + K^2\psi_4(0) \tag{23}$$

The term $\psi_1(0)$ represents the nondimensional mean-square lift on the wing with no alleviation. The remaining terms are associated with alleviation obtained by flap deflection. Since the purpose of a gust-alleviation system is to make the total wing lift as small as possible, it is desirable to select the constant K to minimize $\tilde{\psi}_e(0)$. The proper value of K can be obtained by differentiating equation (23) with respect to K, setting the result equal to zero, and solving for K. The result is

$$K = \frac{\psi_2(0)}{\psi_4(0)} \tag{24}$$

Power spectrum of lift.- The power spectrum of wing lift $e(x)$ is defined as the Fourier transform of the correlation function of $e(x)$; that is,

$$\phi_e(\omega_g) = \frac{1}{\pi}\int_{-\infty}^{\infty} \psi_e(U\tau)\exp(-i\omega_g\tau)d\tau \tag{25}$$

Since the function $\psi_e(U\tau)$ is an even function, equation (25) can be written as

$$\phi_e(\omega_g) = \frac{2}{\pi}\int_0^{\infty} \psi_e(U\tau)\cos(\omega_g\tau)d\tau \tag{26}$$

or, from equation (17),

$$\phi_e\left(C \frac{U}{L}\right) = \frac{2}{\pi}\left(\frac{C_{L_\alpha}qS}{U}\right)^2 \overline{w^2} \frac{L}{U}\int_0^{\infty} \tilde{\psi}_e(L\xi)\cos(C\xi)d\xi \tag{27}$$

where

$$C = \frac{\omega_g L}{U}$$

$$\xi = \frac{U\tau}{L}$$

Inspection of equation (16) shows that equation (27) can be written as the sum of four terms:

$$\tilde{\phi}_e(C) = \phi_1(C) - K\phi_2(C) - K\phi_3(C) + K^2\phi_4(C) \tag{28}$$

where

$$\tilde{\phi}_e(C) = \frac{\phi_e\left(C \frac{U}{L}\right)}{\frac{2}{\pi}\left(\frac{C_{L_\alpha}qS}{U}\right)^2 \overline{w^2} \frac{L}{U}} \tag{29}$$

and

$$\phi_i(C) = \int_0^{\infty} \psi_i(L\xi)\cos(C\xi)d\xi \qquad (i = 1, 2, 3, \text{ or } 4) \tag{30}$$

APPLICATION OF EQUATIONS

In order to compute the correlation function, mean-square values, or power spectrum of lift for a specific configuration, expressions must be assumed for the lift weighting function $\gamma(y)$, the function $g(\sigma)$ associated with isotropic turbulence, and the impulsive flap response $h(\lambda)$.

The expression for $\gamma(y^*)$ used in this study corresponds to an elliptically loaded wing and is given by (ref. 7)

$$\gamma(y^*) = \frac{4}{\pi}\sqrt{1 - (y^*)^2} \tag{31}$$

A form of the function $g(\sigma)$, which has previously been considered in reference 7 in studying isotropic turbulence, is used in this study. For two vertical gust velocities located at the points (x_1,y_1) and (x_2,y_2), this function takes the form

$$g(\sigma) = \exp\left(-2 \frac{\sigma}{L}\right) \tag{32}$$

where

$$\sigma = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(U\tau)^2 + \left(\frac{b}{2}\right)^2\left(y_2{}^* - y_1{}^*\right)^2} \tag{33}$$

The flap response $h(\lambda)$ to an impulse depends on the assumed characteristics of the flap and its drive system and on its relation to the angle sensed by the gust-sensing vane. Equations will be derived for two types of flap response: (1) Instantaneous (in-phase) response and (2) response described by a second-order linear differential equation.

In-Phase Flap Response

When the flaps move in phase with the vane, $h(\lambda)$ becomes a Dirac delta function which is written as

$$h(\lambda) = \delta(\lambda) \tag{34}$$

The correlation function of $e(x)$ is determined by use of equations (31) to (34) and equations (16) to (21). The results are

$$\tilde{\psi}_e(\xi) = \psi_1(\xi) - K\psi_2(\xi) - K\psi_3(\xi) + K^2\psi_4(\xi) \tag{35}$$

where

$$\tilde{\psi}_e(\xi) = \frac{\psi_e(\xi)}{\left(\frac{C_{L_\alpha} qS}{U}\right)^2 \overline{w^2}} \tag{36}$$

$$\psi_1(\xi) = \left(\frac{2}{\pi}\right)^2 \int_{-1}^{1} \int_{-1}^{1} \sqrt{1 - (y_1{}^*)^2} \sqrt{1 - (y_2{}^*)^2} \exp\left[-2\sqrt{\xi^2 + \left(\frac{b'}{2}\right)^2\left(y_2{}^* - y_1{}^*\right)^2}\right] dy_1{}^* \, dy_2{}^* \tag{37}$$

$$\psi_2(\xi) = \frac{4}{\pi}\int_0^1 \sqrt{1 - \left(y_1{}^*\right)^2} \exp\left[-2\sqrt{(\xi + l')^2 + \left(\frac{b'}{2}\right)^2\left(y_1{}^*\right)^2}\right] dy_1{}^* \tag{38}$$

$$\psi_3(\xi) = \frac{4}{\pi}\int_0^1 \sqrt{1 - (y_2{}^*)^2}\, \exp\left[-2\sqrt{(\xi - l')^2 + \left(\frac{b'}{2}\right)^2 (y_2{}^*)^2}\right] dy_2{}^* \tag{39}$$

$$\psi_4(\xi) = \exp(-2|\xi|) \tag{40}$$

Nondimensional mean-square value of lift.- The nondimensional mean-square value of lift is given by equation (23), which reduces to the form

$$\tilde{\psi}_e(0) = \psi_1(0) - 2K\psi_2(0) + K^2 \tag{41}$$

where

$$\psi_1(0) = \left(\frac{2}{\pi}\right)^2 \int_{-1}^1 \int_{-1}^1 \sqrt{1 - (y_1{}^*)^2}\, \sqrt{1 - (y_2{}^*)^2}\, \exp\left[-2\left|\frac{b'}{2}(y_2{}^* - y_1{}^*)\right|\right] dy_1{}^*\, dy_2{}^* \tag{42}$$

and

$$\psi_2(0) = \frac{4}{\pi}\int_0^1 \sqrt{1 - (y_1{}^*)^2}\, \exp\left[-2\sqrt{(l')^2 + \left(\frac{b'}{2}\right)^2 (y_1{}^*)^2}\right] dy_1{}^* \tag{43}$$

Note that the nondimensional mean-square value is determined by the parameters b' and l'.

With the substitution of $\psi_2(0)$ and $\psi_4(0)$ from equations (38) and (40), respectively, into equation (24) for the gain constant K which minimizes the mean-square value of $e(x)$, the following result is obtained:

$$K = \frac{4}{\pi}\int_0^1 \sqrt{1 - (y_1{}^*)^2}\, \exp\left[-2\sqrt{(l')^2 + \left(\frac{b'}{2}\right)^2 (y_1{}^*)^2}\right] dy_1{}^* \tag{44}$$

Nondimensional power spectrum of lift.- The nondimensional power spectrum of lift is computed by using equations (27), (29), and (30).

Effect of Lag in Flap Response

In this study, it is assumed that the flap deflection $\delta_f(x)$ is related to the angle of attack of the gust sensor $\alpha_v(x+l,0)$ by means of a linear second-order differential equation of the form

$$\frac{d^2}{dx^2}\delta_f(x) + 2\zeta\Omega_n \frac{d}{dx}\delta_f(x) + \Omega_n{}^2\delta_f(x) = K_1\Omega_n{}^2\alpha_v(x+l,0) \tag{45}$$

where ζ is the damping ratio, Ω_n is the natural frequency in radians per distance, and K_1 is a gain or proportionality constant.

For a damping ratio $\zeta < 1$, the solution of equation (45) is given by equation (9) in which $h(\lambda)$ is equal to

$$h(\lambda) = \frac{\Omega_n}{\sqrt{1 - \zeta^2}} \exp\left(-\zeta\Omega_n\lambda\right)\sin\left(\Omega_n\sqrt{1 - \zeta^2}\,\lambda\right) \tag{46}$$

Since $x = Ut$, Ω_n can be replaced in equation (46) by the equivalent expression

$$\Omega_n = \frac{\omega_n}{U}$$

With this substitution, equation (46) becomes

$$h(\lambda) = \frac{\omega_n}{U\sqrt{1 - \zeta^2}} \exp\left(-\zeta\,\frac{\omega_n}{U}\,\lambda\right)\sin\left(\frac{\omega_n}{U}\sqrt{1 - \zeta^2}\,\lambda\right) \tag{47}$$

Correlation function of lift.- The correlation function of lift can be determined by substituting equations (31), (32), (33), and (47) into equations (16) to (21), and the result is

$$\tilde{\psi}_e(\xi) = \psi_1(\xi) - K\psi_2(\xi) - K\psi_3(\xi) + K^2\psi_4(\xi) \tag{48}$$

where

$$\tilde{\psi}_e(\xi) = \frac{\psi_e(\xi)}{\left(\frac{C_{L_\alpha}qS}{U}\right)^2 \overline{w^2}} \tag{49}$$

$$\psi_1(\xi) = \left(\frac{2}{\pi}\right)^2 \int_{-1}^{1}\int_{-1}^{1} \sqrt{1 - (y_1{}^*)^2}\,\sqrt{1 - (y_2{}^*)^2}$$

$$\exp\left[-2\sqrt{\xi^2 + \left(\frac{b'}{2}\right)^2 (y_2{}^* - y_1{}^*)^2}\right] dy_1{}^*\,dy_2{}^* \tag{50}$$

$$\psi_2(\xi) = \frac{\frac{4}{\pi}k}{\sqrt{1 - \zeta^2}} \int_0^1 \int_0^\infty \sqrt{1 - (y_1{}^*)^2}\,\exp\left(-\zeta k\lambda_2{}'\right)$$

$$\sin\left(k\sqrt{1 - \zeta^2}\,\lambda_2{}'\right)\exp\left[-2\sqrt{(\xi + l' - \lambda_2{}')^2 + \left(\frac{b'}{2}\right)^2 (y_1{}^*)^2}\right] d\lambda_2{}'\,dy_1{}^* \tag{51}$$

$$\psi_3(\xi) = \frac{\frac{4}{\pi}k}{\sqrt{1 - \xi}} \int_0^1 \int_0^\infty \sqrt{1 - \left(y_2^*\right)^2} \exp(-\xi k\lambda_1')\sin\left(k\sqrt{1 - \xi^2}\,\lambda_1'\right)$$

$$\exp\left[-2\sqrt{\left(\xi - l' + \lambda_1'\right)^2 + \left(\frac{b'}{2}\right)^2 \left(y_2^*\right)^2}\right] d\lambda_1'\,dy_2^* \tag{52}$$

$$\psi_4(\xi) = \left(\frac{k}{\sqrt{1 - \xi}}\right)^2 \int_0^\infty \int_0^\infty \exp\left[-k\xi\left(\lambda_1' + \lambda_2'\right)\right]\sin\left(k\sqrt{1 - \xi^2}\,\lambda_1'\right)$$

$$\sin\left(k\sqrt{1 - \xi^2}\,\lambda_2'\right)\exp\left(-2\left|\xi + \lambda_1' - \lambda_2'\right|\right)d\lambda_1'\,d\lambda_2' \tag{53}$$

Mean-square value of lift.- The mean-square value of lift can be determined by evaluating equations (48) to (53) with ξ equal to zero.

Power spectrum of lift.- The power spectrum of lift is determined by substituting equations (50) to (53) into equations (28), (29), and (30) and integrating the resulting expressions.

RESULTS AND DISCUSSION

A basic criterion used in this study to evaluate the effectiveness of the alleviation system is the reduction in the mean-square value of wing lift $e(x)$. The mean square of $e(x)$ is discussed, then the power spectrum of $e(x)$ is used to examine how this reduction in mean-square lift is distributed over the different gust frequencies.

The various integrals in this study were evaluated numerically on a digital computer.

In-Phase Flap Response

Nondimensional mean-square lift.- Figure 3 shows the nondimensional mean-square lift $\tilde{\psi}_e(0)$ due to gusts acting on a wing flying through isotropic turbulence with and without the gust alleviation system. The nondimensional mean-square lift is presented as a function of b', the ratio of wing span to the scale of turbulence, and l^*, the vane location in fractions of wing span.

Note that the nondimensional mean-square lift on the basic wing (no alleviation) is reduced as the wing span becomes larger relative to the scale of turbulence (b' increases). This self-alleviation is due to a spanwise averaging effect; that is, the lift increments due to various gust velocities tend to average out.

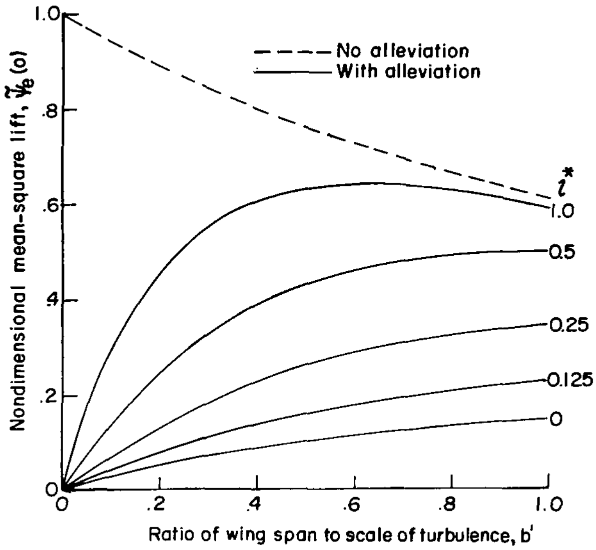


Figure 3.- Effect of alleviation system on the mean-square value of $e(x)$. (Flap moving in phase with vane and optimum value of gain constant K used.)

The solid lines represent the results for the flaps moving in phase with the vane, that is, when there is no lag in the system. The vane senses the gust and the flaps respond immediately. The values of K used to obtain these curves were those which minimized the mean-square lift, as determined from equation (44).

A wide range of b' values are shown in the figure to clarify the various trends; however, practical values of b' appear to be rather small – on the order of 0.1 or less. For the flight tests in references 2 and 3, the vane location l^* appears to be about 0.5. At this value of l^*, $\tilde{\psi}_e(0)$ is reduced from 1 to 0, or 100 percent, at $b' = 0$ and from 0.94 to 0.14, or 85 percent, at $b' = 0.1$. Thus, for practical values of b' up to 0.1, the figure shows that the nondimensional mean-square lift should be reduced by 85 to 100 percent, provided the response of the servo driving the flaps is fast enough.

Gain constant K.- The results of figure 3 were generated by choosing a value of K to minimize the mean-square value of lift on the alleviated wing. The variation of gain constant with b' and l^* is shown in figure 4. Note that as the wing span and vane location ahead of the wing increase, K decreases. This would be expected since the gust velocity measured by a vane far ahead of the wing would not be expected to be useful in predicting the present gust velocity on the wing; hence, the optimum value of K would approach zero as l^* approaches infinity. Likewise, as the wing span increases, the gust velocities on the wing could differ appreciably from the gust velocity measured by the vane at a point, and the optimum value of K would again approach zero; however, it turns out that the wing is alleviating itself in this case as shown previously in figure 3. For practical values of b', the gain constant varies from about 0.8 to 1.0.

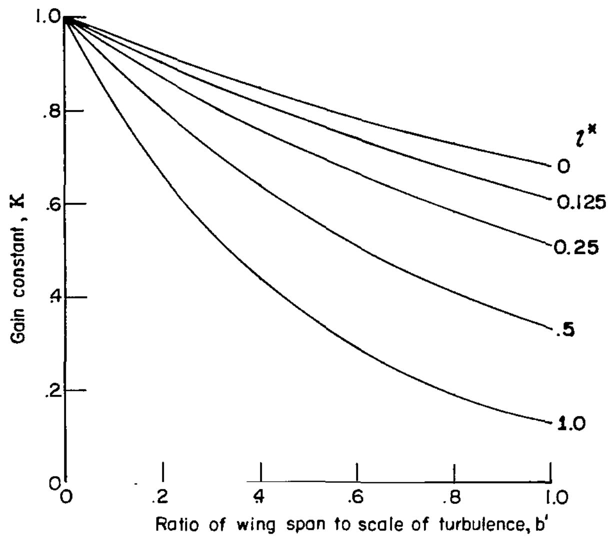


Figure 4.- Gain constant used to minimize the mean-square value of e(x). (Flap moving in phase with vane.)

Power spectrum of lift.- It is of interest to examine the reduction in the power spectrum over the gust frequency range which results from minimizing the mean square of e(x). The nondimensional power spectrum of the lift on the basic wing is plotted against the nondimensional gust frequency C on a log-log scale in figure 5 for b' values of 0.05, 0.10, and 1.0. As expected, the power spectrum of wing lift is much more sensitive to the lower gust frequency and decreases with increase in gust frequency. Note that the power spectrum is relatively insensitive to small b' values except at the larger frequencies.

Figure 6 gives the percent reduction in the power spectra of figure 5 when the alleviation system is employed. A value of $R = 100$ indicates complete alleviation. Frequencies at which $R = 0$ on any of the curves indicate the point where the power associated with the alleviated wing first becomes larger than the power associated with the unalleviated wing. For any value of l^*, there is a reduction in alleviation as gust frequency increases. This results because the high-frequency components of gust velocity over the wing span are not highly correlated with the velocity sensed by the vane. The correlation becomes worse as l^* increases.

Figure 6 shows that the gust frequency range over which continuous alleviation is obtained (that is, $R > 0$) decreases with increase in wing size and vane location ahead of the wing. In all cases, however, the greatest reductions occur at the lower frequencies, which are more important for passenger comfort.

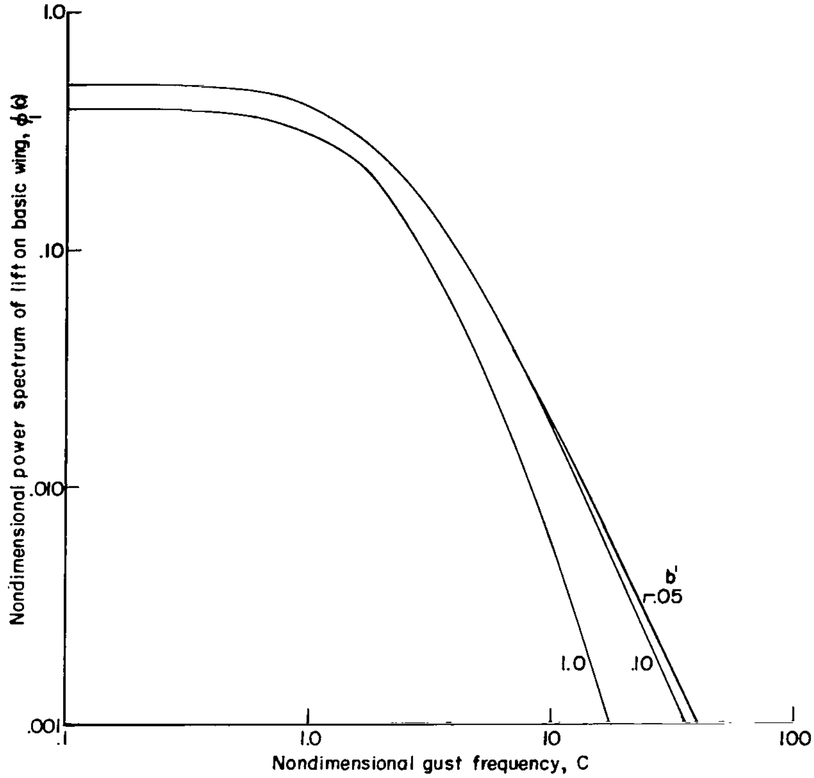
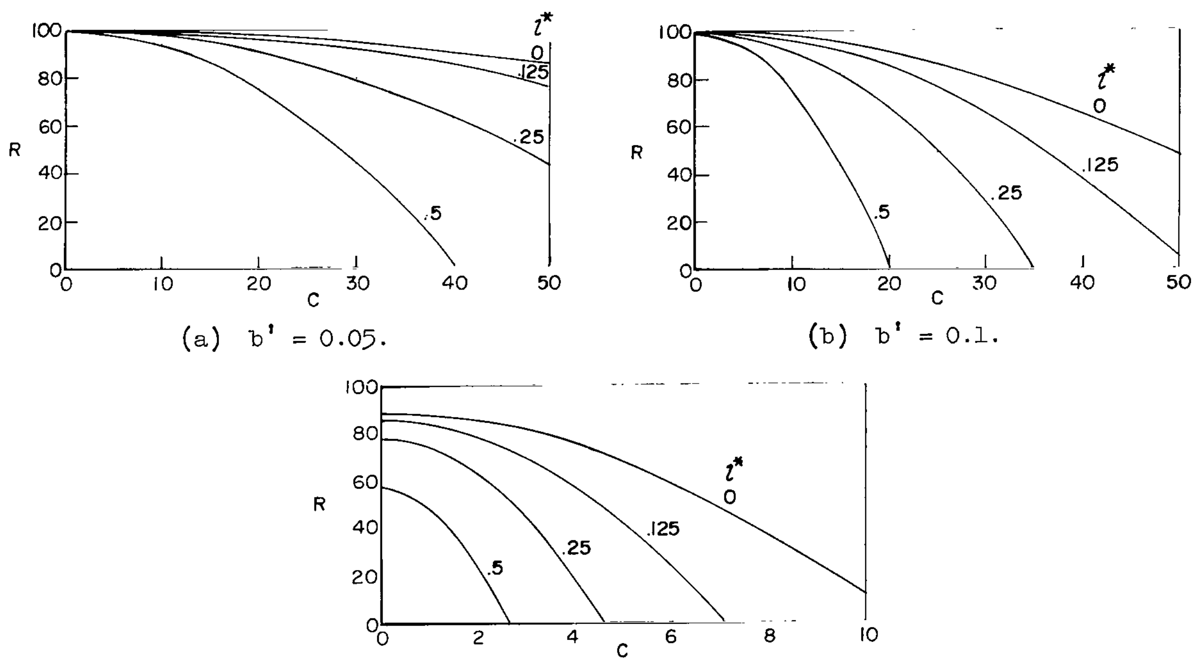


Figure 5.- Power spectrum of lift on basic wing.



(a) b' = 0.05.

(b) b' = 0.1.

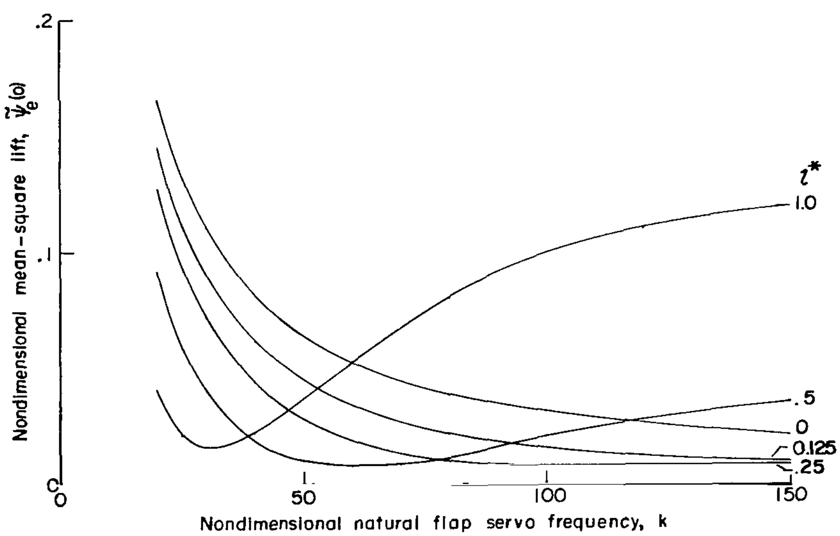
(c) b' = 1.0.

Figure 6.- Percentage reduction in power spectrum of lift on basic wing when the alleviation system is used. (Flap moving in phase with vane.)

Lag Effects in Flap Response

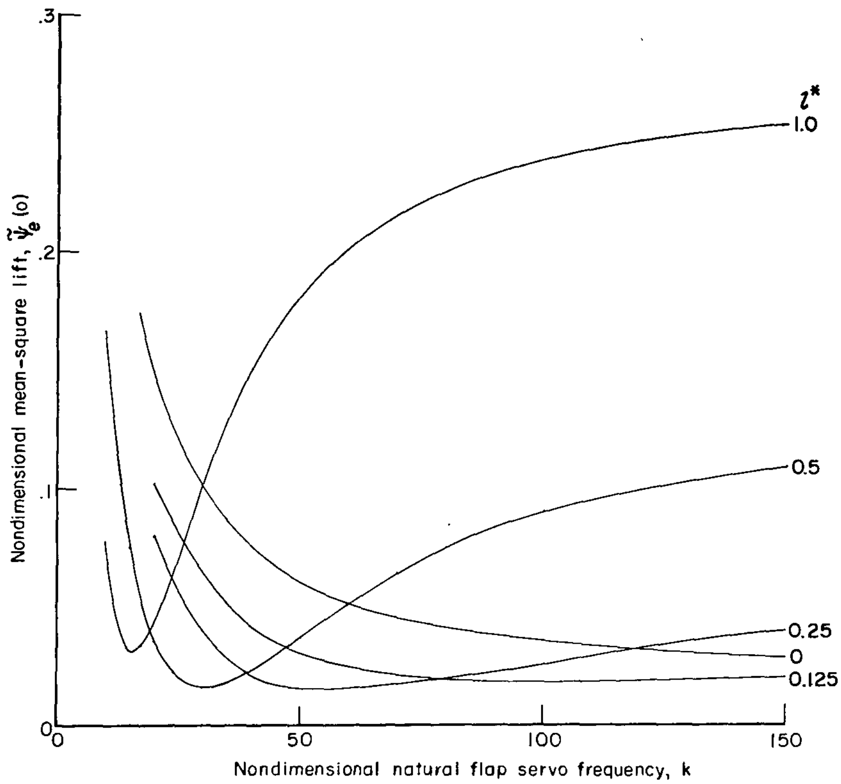
Notice that as the vane is moved farther ahead of the wing in figure 3 (that is, l^* increases); the gust alleviation becomes less. The reason for this is that the flaps move too soon. If the vane is mounted ahead of the wing, there should be a time lag in the response of the flap to allow the gust sensed by the vane to reach the wing. This time lag can be incorporated into the system by varying the frequency of the servosystem driving the flaps. The present study always uses a damping ratio ζ of 0.7, which means a response with small overshoot.

Nondimensional mean-square lift.- The nondimensional mean-square lift $\tilde{\psi}_e(0)$ is presented as a function of the nondimensional natural frequency k of the flap servo in figure 7 for b' values of 0.05, 0.1, and 1.0. This figure shows that there are optimum servo frequencies for minimizing the values of mean-square lift for given values of l^* and b'. For example, if $b' = 0.05$, then the curve in figure 7(a) corresponding to $l^* = 0.5$ shows that the optimum value of k for reducing the nondimensional mean-square lift is about 61. As k increases, the values of mean-square lift $\tilde{\psi}_e(0)$ approach the in-phase values shown in figure 3 for the particular values of l^* and b' under consideration.

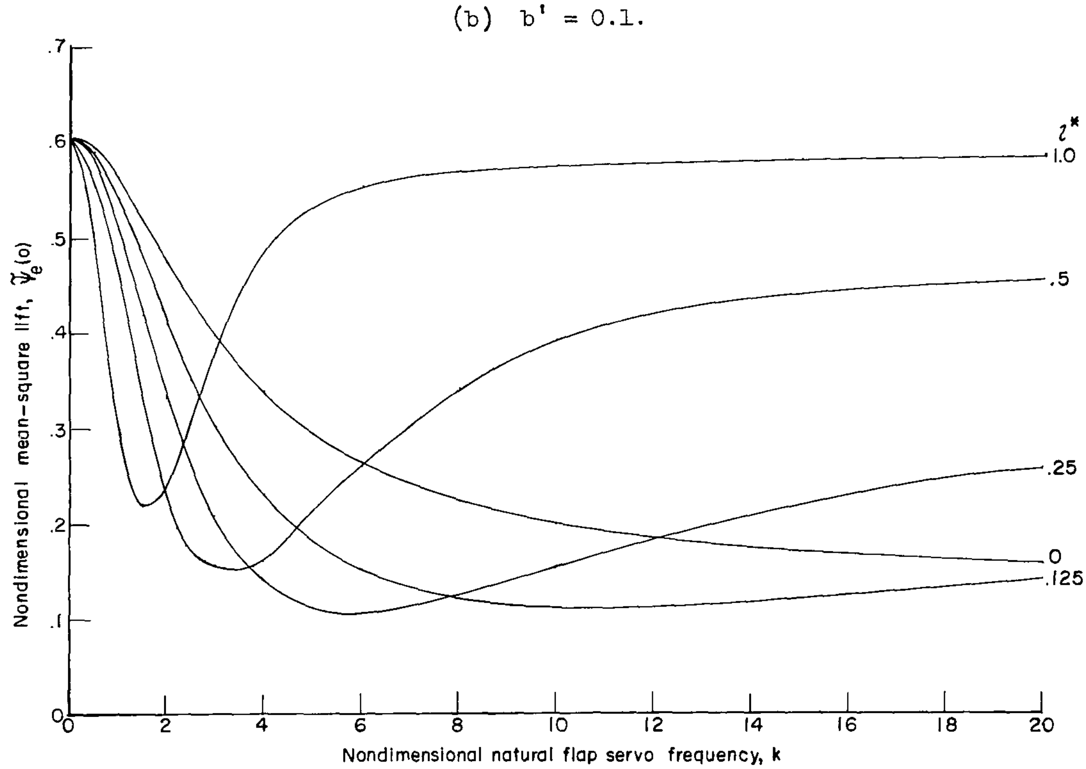


(a) $b' = 0.05$.

Figure 7.- Effect of servo response of the alleviation system on the mean-square value of $e(x)$. $\zeta = 0.7$.



(b) b' = 0.1.



(c) b' = 1.0.

Figure 7.- Concluded.

Gain constant K.- Figure 8 gives the values of K used to generate figure 7. As k increases, the gain constant K approaches the in-phase values shown in figure 4 for the particular values of l^* and b' under consideration.

Optimum values of k and K.- Figure 9 presents the optimum values of servo frequency k and gain constant K required to minimize the nondimensional mean-square lift $\tilde{\psi}_e(0)$ for different values of l^* and b'. The resulting percentage reduction in the mean-square values of lift on the basic wing when these optimum values of k and K are used is shown in figure 10.

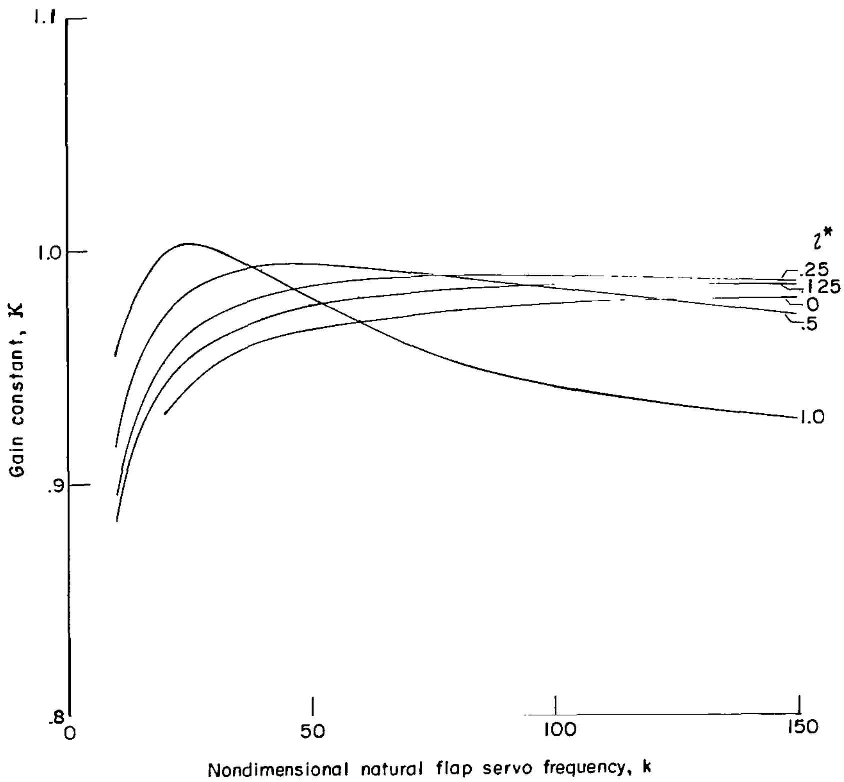
As an example in the use of figures 9 and 10, consider the following specific values:

$b = 15.24$ meters (50 ft)

$U = 60.96$ meters/sec (200 ft/sec)

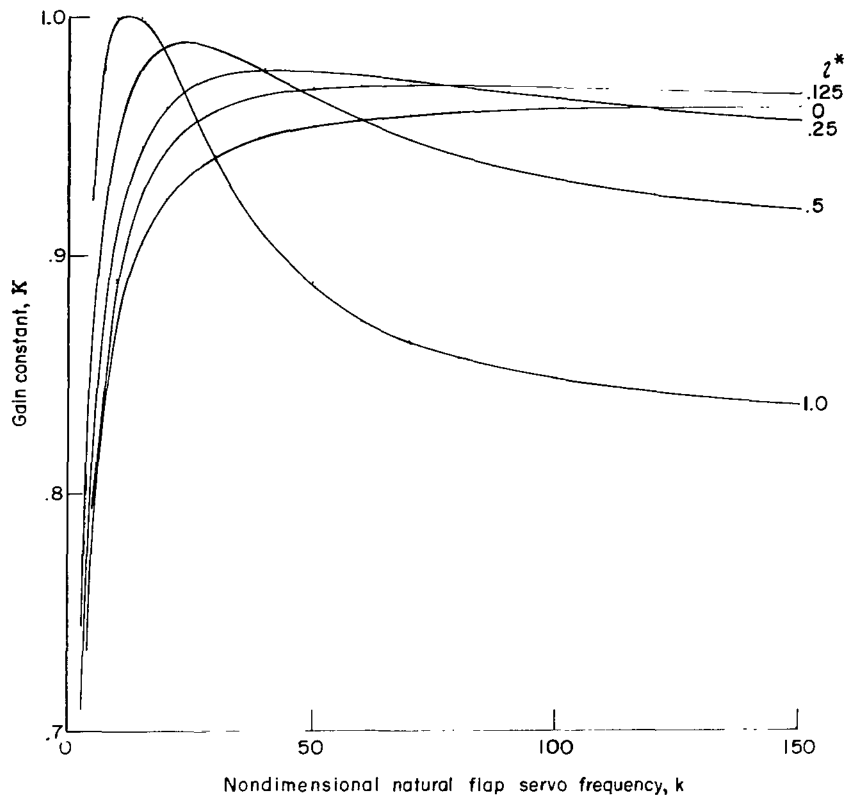
$l = 7.62$ meters (25 ft)

$L = 304.8$ meters (1000 ft)

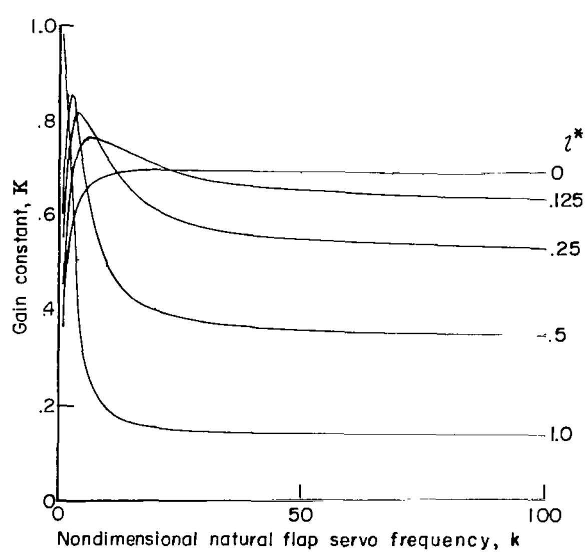


(a) $b' = 0.05$.

Figure 8.- Gain constant used to minimize the mean-square value of $e(x)$.



(b) $b' = 0.1$.



(c) $b' = 1.0$.

Figure 8.- Concluded.

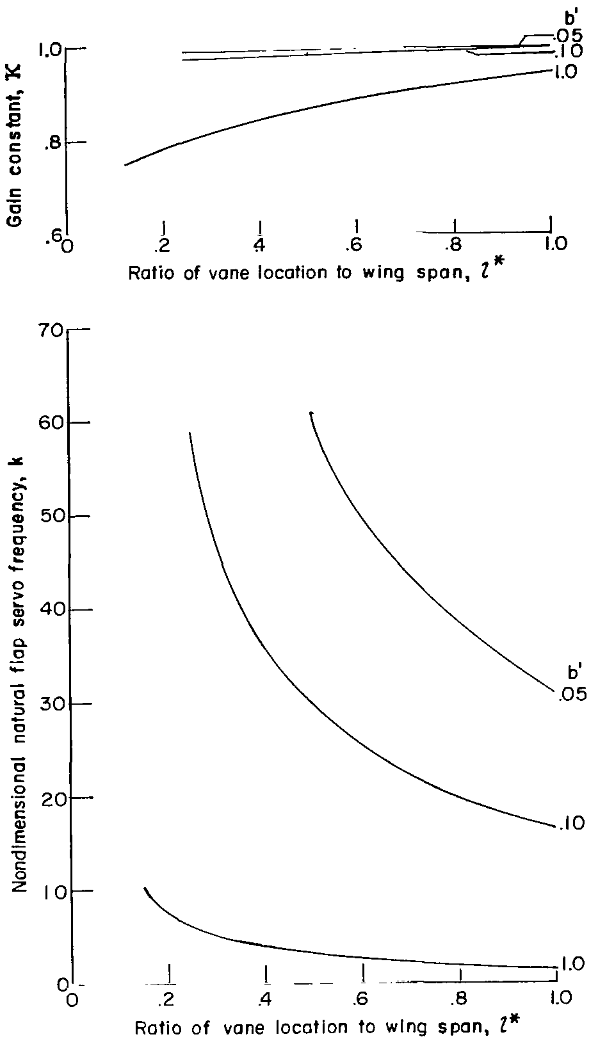


Figure 9.- Optimum values of K and k to minimize the mean-square value of e(x).

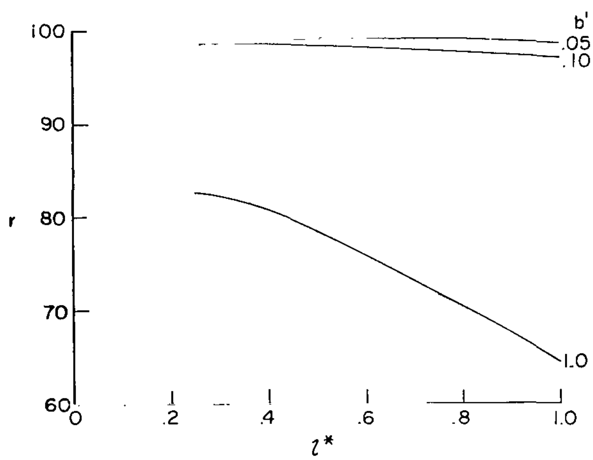


Figure 10.- Percentage reduction in mean-square lift on the basic wing when optimum values of k and K are used in the flap servo system.

Then,

$$l^* = 0.5$$

$$b' = 0.05$$

$$\frac{L}{U} = 5 \text{ sec}$$

From figure 9, the optimum value of k for this value of l^* and b' is about 61. From the definition of k, the dimensional natural frequency of the flap servosystem ω_n is related to k as

$$\omega_n = \frac{k}{2\pi\left(\frac{L}{U}\right)}$$

where the factor 2π is introduced so that ω_n can be expressed in Hz (or cycles per second).

Thus, the optimum natural frequency of the flap servo in the example is

$$\omega_n = \frac{61}{2\pi(5)} = 1.94 \text{ Hz}$$

The optimum gain constant K shown in figure 9 is very close to 1. Furthermore, the reduction in the mean-square lift, given by figure 10, is about 99 percent.

A Specific Flap Control System

By definition, the natural frequency ω_n corresponding to an optimum value of k depends on the unknown scale of turbulence L. In addition, the atmosphere is isotropic in patches at most and the scale of turbulence changes. Hence, it might be better not to try to optimize ω_n and K for a specific value of L, but rather to choose constant values of ω_n and K so that over a given range of possible L values, acceptable alleviation results will be obtained. Intuitively, it would appear that if such a system were feasible, the system would also function well in an atmosphere which is not quite isotropic.

A fixed practical set of control parameters which were chosen somewhat arbitrarily for investigation are

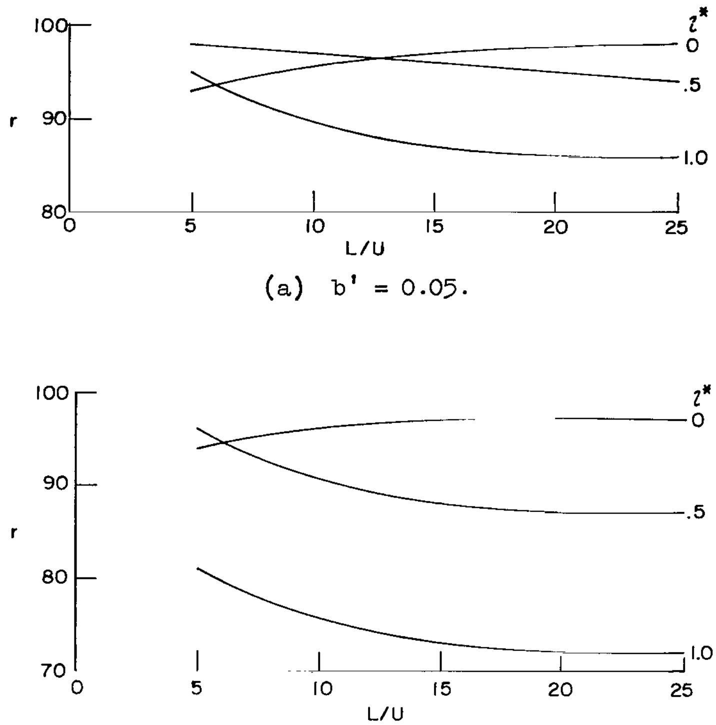
$$\omega_n = 1.6 \text{ Hz}$$

$$K = 0.9$$

$$\zeta = 0.7$$

Figure 11 shows the percentage reduction in the mean-square lift of the basic wing when this alleviation system is used for b' values of 0.05 and 0.1. The percentage reduction is presented as a function of the ratio L/U and vane location l^*. The point of figure 11 is that the reduction is very substantial when the constant control parameters are used, and the unknown value of L/U falls within the wide range of values shown.

For the airplane used in the flight tests of references 2 and 3, the wing span b was about 15.24 meters (50 feet) and the forward velocity U was about 60.96 meters/sec (200 ft/sec). Calculations were made to determine the reduction in the power spectrum of lift of the basic wing in this case. It was found that the results varied only slightly for reasonable values of L. The reason for this is that the wing span is much smaller than the scale of turbulence. The reduction in the power is shown in figure 12 as a function of gust frequency for different vane locations. The vane location for the flight tests (refs. 2



(a) b' = 0.05.

(b) b' = 0.1.

Figure 11.- Percent reduction in mean-square lift on the basic wing when specific flap control system is used. $\zeta = 0.7$; $\omega_n = 1.6$ Hz; K = 0.9.

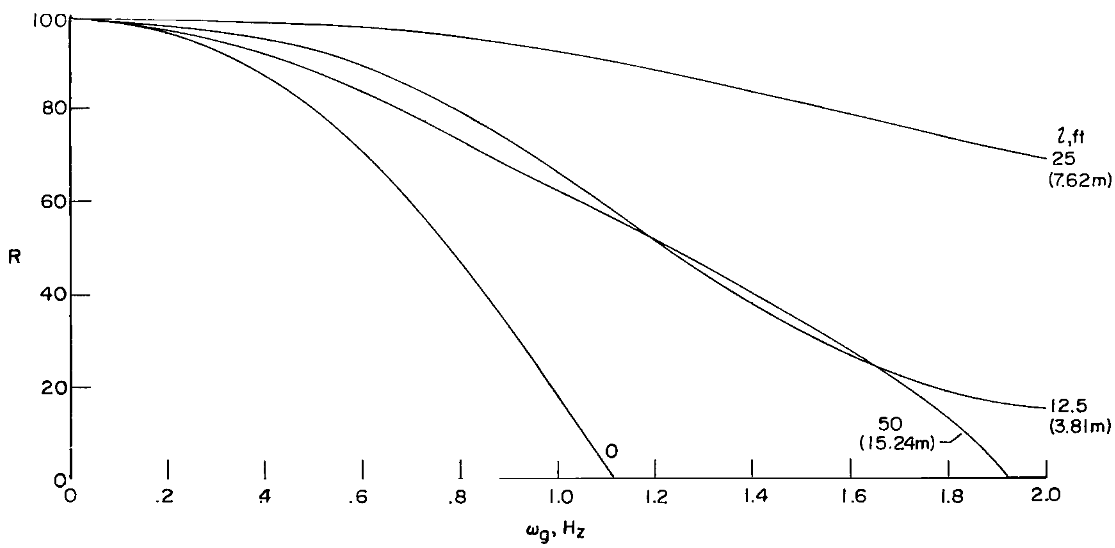


Figure 12.- Reduction in power spectrum of lift on the basic wing when specific flap control system is used. $\zeta = 0.7$; $\omega_n = 1.6$ Hz; K = 0.9.

and 3) was about $l = b/2 = 7.62$ meters (25 ft). For this vane location, the power is reduced 100 percent at low frequencies and 70 percent at 2 Hz, which was considered to be the highest frequency of interest in the flight tests from the standpoint of passenger comfort. Since the alleviation system becomes less effective as the gust frequency increases, a low-band-pass filter should be used to attenuate the alleviation system beyond the frequency range of interest. Hence, it appears that if the frequency of the flap system is about 1.6 Hz, very effective gust alleviation can be realized for the type of aircraft used in the flight tests.

In this study, the statistical properties of the lift on an airplane wing equipped with a gust alleviation system in flight through isotropic turbulence have been considered. A reduction in the wing lift due to the gusts by the alleviation system is, however, only part of the problem. A complete analysis should include the dynamic response of the entire airplane to the gust as was done, for example, in reference 1. Consideration of the spanwise variation of gusts has, however, indicated design parameters (flap servo gain and natural frequency and vane location) that give substantial reduction of the lift due to gusts.

CONCLUDING REMARKS

An analysis has been made of the effects of spanwise variations of gust velocity in isotropic turbulence on a gust alleviation system which employs an angle-of-attack vane mounted ahead of the wing to sense the vertical gust velocity. The wing flaps were moved in response to the vane deflection by a linear second-order servosystem to produce a lift opposite to that produced by the gust. Since the gust velocity actually varies across the wing span and the vane only senses the gust velocity at a single point, the flaps cannot be expected to eliminate the effects of the gusts completely by being deflected in response to a signal from the vane. The effectiveness of the alleviation system is examined by considering the statistical properties of the residual lift on the alleviated wing. Consideration of spanwise variation of gusts has indicated design parameters (gain and natural frequency of the flap servosystem and vane location) that give substantial reduction of the lift due to gusts.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., February 11, 1971.

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